Unit FE-2 Foundation Electricity: DC Network Analysis

What this unit is about

This unit contains some basic ideas on DC network analysis. It also deals with the Thevenin theorem, a technique of considerable use in power systems engineering.

Why is this knowledge necessary?

Renewable energy sources are used mainly to generate electrical power, which is injected into power networks consisting of a large number of transmission lines, other conventional generators and consumers. Such power networks, especially in developed countries, are of considerable complexity. To determine the way these injected powers flow from generators to consumers requires complex calculations based on network analysis. This Unit provides background material for the development of AC circuit theory and of the mathematical expressions that define the power flows in a network. The material in the whole of this unit is required as background knowledge for all the other Foundation Units on electricity (FE-3 to FE-5).

At the beginning of each section the course module(s) that requires the material in this particular section as background knowledge are indicated in bold italics.

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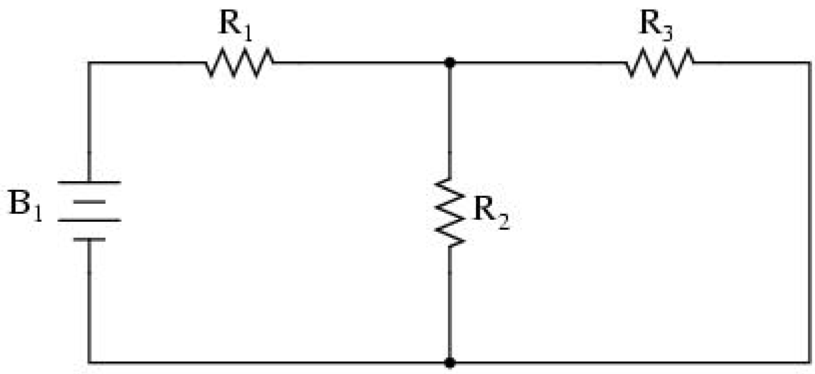
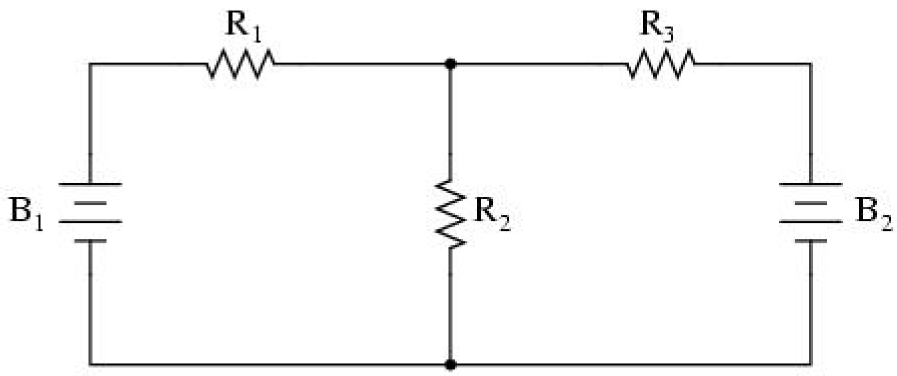
1. What is network analysis?

Generally speaking, *network analysis* is any structured technique used to mathematically analyse a circuit (a "network" of interconnected components). Quite often a power systems engineer will encounter circuits containing multiple sources of power or component configurations which defy simplification by series/parallel analysis techniques. In those cases, he or she will be forced to use other means. This Unit presents a few techniques useful in analysing such complex circuits.
To illustrate how even a simple circuit can defy analysis by breakdown into series and parallel portions let us examine this series-parallel circuit:



To analyse the above circuit, one would first find the equivalent of R_2 and R_3 in parallel, then add R_1 in series to arrive at a total resistance. Then, taking the voltage of battery B_1 with that total circuit resistance, the total current could be calculated through the use of Ohm's Law (I=E/R). This current figure will then be used to calculate voltage drops in the circuit. All in all, a fairly simple procedure.
However, the addition of just one more battery could change all of that:



Resistors R_2 and R_3 are no longer in parallel with each other, because B_2 has been inserted into R_3's branch of the circuit. Upon closer inspection, it appears there are *no* two resistors in this circuit directly in series or parallel with each other. This is the crux of our problem: in series-parallel analysis, we started off by identifying sets of resistors that *were* directly in series or parallel with each other, and then reduce them to single, equivalent resistances. If there are no resistors in a simple series or parallel configuration with each other, then what can we do?

Although it might not be apparent at this point, the heart of the problem is the existence of multiple unknown quantities. With these problems, more than one variable is unknown at the most basic level of circuit simplification therefore to determine the variables we have to solve a set of simultaneous equations.
A number of techniques have been developed to determine the currents in the branches of complex electrical networks when voltage and/or current sources are connected to the network nodes. It is very likely that in your school Physics course or at University you have met techniques such as the branch-current and the loop-current methods. In the MSc course we are primarily interested in energy transfer from sources to consumers. Hence we only need the basic Kirchhoff's laws discussed in FE-1 to help us derive AC power transfer expressions in Unit FE-3 and subsequently in the 'Integration' module. As a consequence there is no need to cover in detail such network techniques, although students who are interested could find useful information in the references given at the end of this Unit.

Unless you have pursued a degree in electrical engineering, it is unlikely that you would have met Thevenin's theorem, a most powerful technique that it is of particular use in power systems engineering. This network theorem is discussed next.

Section Review:

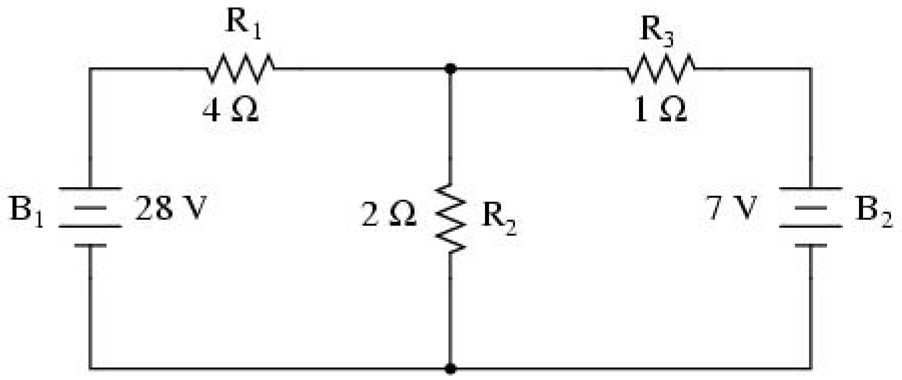
- Some circuit configurations ("networks") cannot be solved by reduction according to series/parallel circuit rules, due to multiple unknown values.
- Mathematical techniques to solve for multiple unknowns can be applied to basic Laws of circuits to solve networks.

2. Thevenin's Theorem

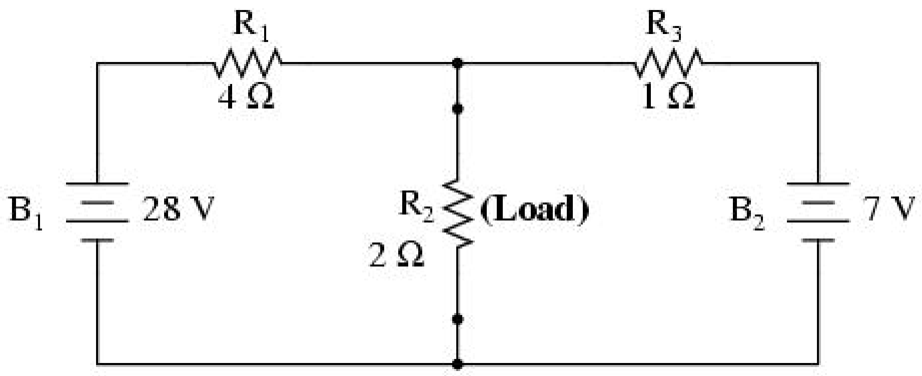
[This material relates predominantly to modules ELP032, ELP033, ELP040]

In electric network analysis, the fundamental rules are Ohm's Law and Kirchhoff's Laws. While these laws may be applied to analyse just about any circuit configuration (even if we have to resort to complex algebra to handle multiple unknowns), there are some "shortcut" methods of analysis.

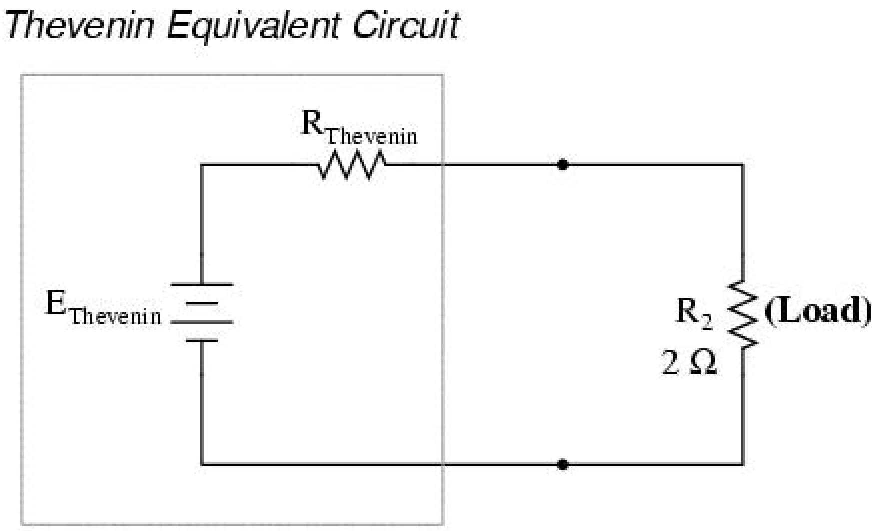
Thevenin's Theorem states that it is possible to simplify any linear circuit, no matter how complex, to an equivalent circuit with just a single voltage source and series resistance connected to a load. The qualification of "linear" excludes some components such as some semiconductor devices, which are nonlinear.
Thevenin's Theorem is especially useful in analysing power systems and other circuits where one particular resistor in the circuit (called the "load" resistor) is subject to change, and re-calculation of the circuit is necessary with each trial value of load resistance, to determine voltage across it and current through it. Let's take another look at our example circuit:



Let's suppose that we decide to designate R_2 as the "load" resistor in this circuit.
Thevenin's Theorem makes this easy by temporarily removing the load resistance from the original circuit and reducing what's left to an equivalent circuit composed of a single voltage source and series resistance. The load resistance can then be re-connected to this "Thevenin equivalent circuit" and calculations carried out as if the whole network were nothing but a simple series circuit:



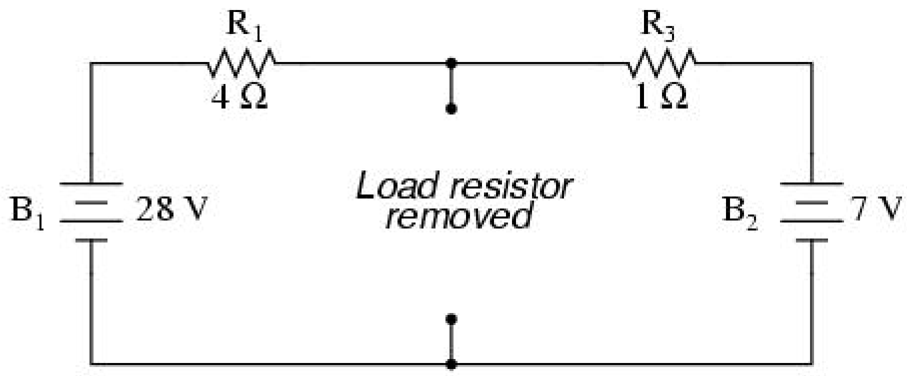
. . . after Thevenin conversion . . .



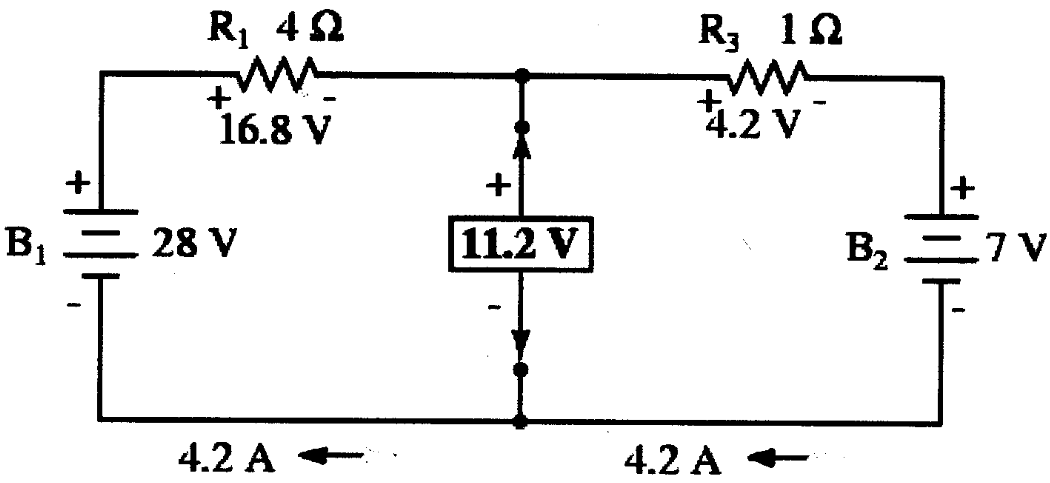
The "Thevenin Equivalent Circuit" is the electrical equivalent of B_1, R_1, R_3, and B_2 as seen from the two points where our load resistor (R_2) connects.

The Thevenin equivalent circuit, if correctly derived, will behave identically to the original circuit formed by B_1, R_1, R_3, and B_2. In other words, the load resistor (R_2) voltage and current should be exactly the same for the same value of load resistance in the two circuits. The load resistor R_2 cannot "tell the difference" between the original network of B_1, R_1, R_3, and B_2, and the Thevenin equivalent circuit of $E_{Thevenin}$, and $R_{Thevenin}$, provided that the values for $E_{Thevenin}$ and $R_{Thevenin}$ have been calculated correctly.

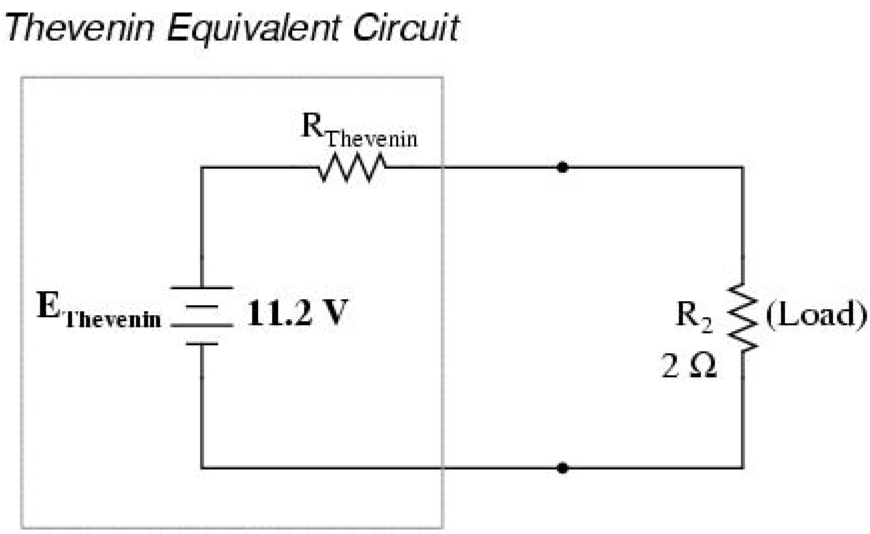
The advantage in performing the "Thevenin conversion" to the simpler circuit, of course, is that it makes load voltage and load current so much easier to solve than in the original network. Calculating the equivalent Thevenin source voltage and series resistance is actually quite easy. First, the chosen load resistor is removed from the original circuit, replaced with a break (open circuit):



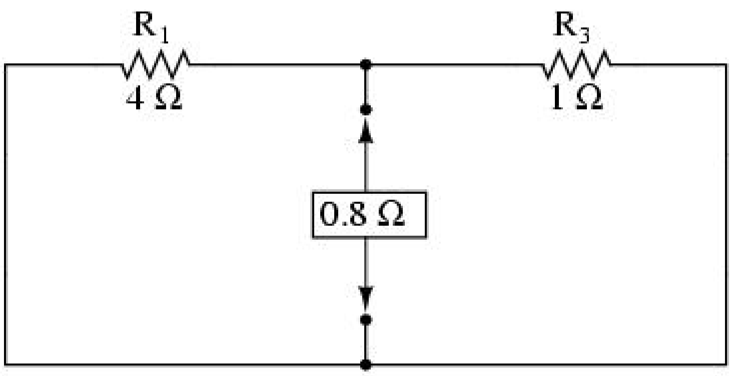
Next, the voltage between the two points where the load resistor used to be attached is determined. Use whatever analysis methods are at your disposal to do this. In this case, the original circuit with the load resistor removed is nothing more than a simple series circuit with opposing batteries, and so we can determine the voltage across the open load terminals by applying the rules of series circuits, Ohm's Law, and Kirchhoff's Voltage Law:



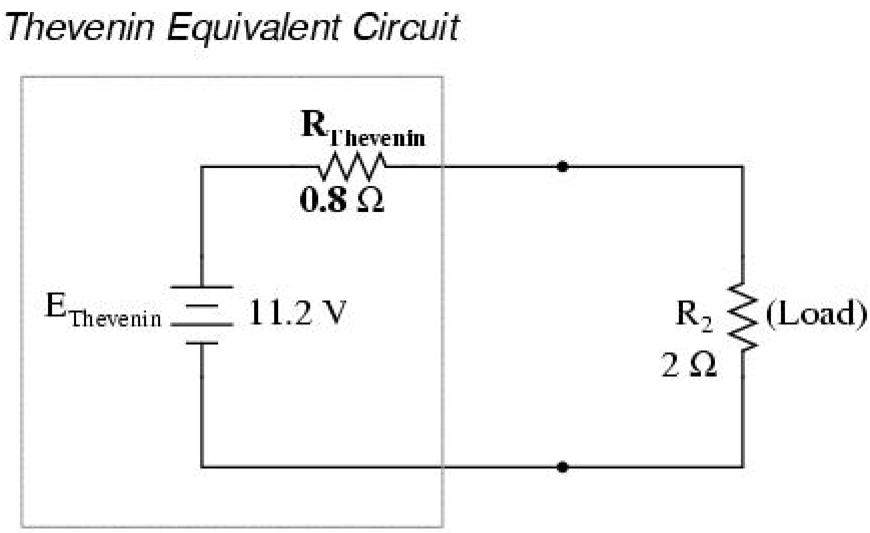
The voltage between the two load connection points can be figured from the one of the battery's voltage and one of the resistor's voltage drops, and comes out to 11.2 volts. This is our "Thevenin voltage" ($E_{Thevenin}$) in the equivalent circuit:



To find the Thevenin series resistance for our equivalent circuit, we need to take the original circuit (with the load resistor still removed), remove and replace by a short circuit the power sources and figure the resistance from one load terminal to the other:



With the removal of the two batteries, the total resistance measured at this location is equal to R_1 and R_3 in parallel: 0.8 Ω. This is our "Thevenin resistance" ($R_{Thevenin}$) for the equivalent circuit:



With the load resistor (2 Ω) attached between the connection points, we can determine voltage across it and current through it as though the whole network were nothing more than a simple series circuit:

$$I = 11.2/(0.8+2) = 4A$$

Notice that the voltage and current figures for R_2 (8 volts, 4 amps) are identical to those found using other methods of analysis. Also notice that the voltage and current figures for the Thevenin series resistance and the Thevenin source (*total*) do not apply to any component in the original, complex circuit. Thevenin's Theorem is only useful for determining what happens to a *single* resistor in a network: the load.
The advantage, of course, is that you can quickly determine what would happen to that single resistor if it were of a value other than 2 Ω without having to go through a lot of analysis again. Just plug in that other value for the load resistor into the Thevenin equivalent circuit and a little bit of series circuit calculation will give you the result.

The Thevenin theorem is of particular use in the calculation of short-circuit currents in power systems. The importance of such calculations and the relevance to the connection of renewable energy sources to power networks will be explored in the module on Integration.

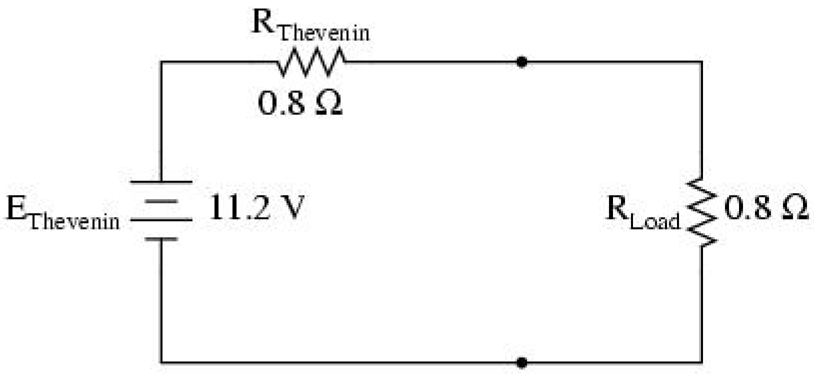
Section Review:

- Thevenin's Theorem is a way to reduce a network to an equivalent circuit composed of a single voltage source, series resistance, and series load.
- Steps to follow for Thevenin's Theorem:
  (1) Find the Thevenin source voltage by removing the load resistor from the original circuit and calculating voltage across the open connection points where the load resistor used to be.
  (2) Find the Thevenin resistance by removing all power sources in the original circuit (voltage sources shorted and current sources open) and calculating total resistance between the open connection points.
  (3) Draw the Thevenin equivalent circuit, with the Thevenin voltage source in series with the Thevenin resistance. The load resistor reattaches between the two open points of the equivalent circuit.
  (4) Analyse voltage and current for the load resistor following the rules for series circuits.

3. Maximum Power Transfer Theorem

The Maximum Power Transfer Theorem is not so much a means of analysis as it is an aid to system design. Simply stated, the maximum amount of power will be dissipated by a load resistance when that load resistance is equal to the Thevenin resistance of the network supplying the power. If the load resistance is lower or higher than the Thevenin resistance of the source network, its dissipated power will be less than maximum.

This is essentially what is aimed for in photovoltaic system design, where the load resistance is matched to the PV array 'source' resistance for maximum power output. Taking our Thevenin equivalent example circuit, the Maximum Power Transfer Theorem tells us that the load resistance resulting in greatest power dissipation is equal in value to the Thevenin resistance (in this case, 0.8 Ω):



With this value of load resistance, the dissipated power will be 39.2W:

If we were to try a lower value for the load resistance (0.5 Ω instead of 0.8 Ω, for example), our power dissipated by the load resistance would decrease to 37.11W: Likewise, if we increase the load resistance (1.1 Ω instead of 0.8 Ω, for example), power dissipation at 38.22W will again be less than it was at 0.8 Ω exactly. (Do these calculations to convince yourself).

Section Review:

- The *Maximum Power Transfer Theorem* states that the maximum amount of power will be dissipated by a load resistance if it is equal to the Thevenin resistance of the network supplying power.

References

'Circuits, Devices and Systems' Smith R.A. & Dorf R.C. 5th edition, John Wiley & Sons. An excellent book on basic electrical engineering. For circuit analysis and network theorems see pp33-57